1.4 Prime Factorization

Essential Question Without dividing, how can you tell when a number is divisible by another number?

1 ACTIVITY: Finding Divisibility Rules for 2, 3, 5, and 10

Work with a partner. Copy the set of numbers (1–50) as shown.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

a. Highlight all the numbers that are divisible by 2.

b. Put a box around the numbers that are divisible by 3.

c. Underline the numbers that are divisible by 5.

d. Circle the numbers that are divisible by 10.

e. **STRUCTURE** In parts (a)–(d), what patterns do you notice? Write four rules to determine when a number is divisible by 2, 3, 5, and 10.

Common Factors and Multiples

In this lesson, you will

- use divisibility rules to find prime factorizations of numbers.

2 ACTIVITY: Finding Divisibility Rules for 6 and 9

Work with a partner.

a. List ten numbers that are divisible by 6. Write a rule to determine when a number is divisible by 6. Use a calculator to check your rule with large numbers.

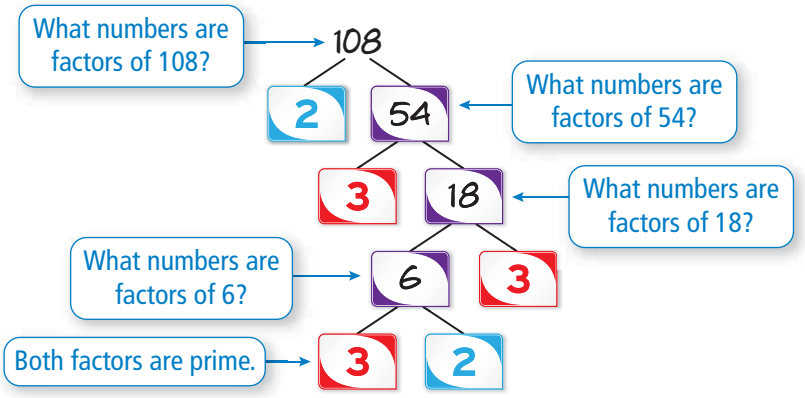
b. List ten numbers that are divisible by 9. Write a rule to determine when a number is divisible by 9. Use a calculator to check your rule with large numbers.

3 ACTIVITY: Rewriting a Number Using 2s, 3s, and 5s

Work with three other students. Use the following rules and only the prime factors 2, 3, and 5 to write each number below as a product.

- Your group should have four sets of cards: a set with all 2s, a set with all 3s, a set with all 5s, and a set of blank cards. Each person gets one set of cards.
- Begin by choosing two cards to represent the given number as a product of two factors. The person with the blank cards writes any factors that are not 2, 3, or 5.
- Use the cards again to represent any number written on a blank card as a product of two factors. Continue until you have represented each handwritten card as a product of two prime factors.
- You may use only one blank card for each step.

a. Sample: 108



$108 = 2 \cdot 3 \cdot 3 \cdot 3 \cdot 2$

Math Practice

Interpret Results

How do you know your answer makes sense?

b. 80 **c.** 162 **d.** 300

e. Compare your results with those of other groups. Are your steps the same for each number? Is your final answer the same for each number?

What Is Your Answer?

4. **IN YOUR OWN WORDS** Without dividing, how can you tell when a number is divisible by another number? Give examples to support your explanation.

5. Explain how you can use your divisibility rules from Activities 1 and 2 to help with Activity 3.

Practice Use what you learned about divisibility rules to complete Exercises 4–7 on page 28.

1.4 Lesson

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Because 2 is factor of 10 and $2 \cdot 5 = 10$, 5 is also a factor of 10. The pair 2, 5 is called a **factor pair** of 10.

EXAMPLE 1 Finding Factor Pairs

Key Vocabulary
factor pair, *p. 26*
prime factorization, *p. 26*
factor tree, *p. 26*

The brass section of a marching band has 30 members. The band director arranges the brass section in rows. Each row has the same number of members. How many possible arrangements are there?

Use the factor pairs of 30 to find the number of arrangements.

$30 = 1 \cdot 30$	There could be 1 row of 30 or 30 rows of 1.
$30 = 2 \cdot 15$	There could be 2 rows of 15 or 15 rows of 2.
$30 = 3 \cdot 10$	There could be 3 rows of 10 or 10 rows of 3.
$30 = 5 \cdot 6$	There could be 5 rows of 6 or 6 rows of 5.
$30 = 6 \cdot 5$	The factors 5 and 6 are already listed.

There are 8 possible arrangements: 1 row of 30, 30 rows of 1, 2 rows of 15, 15 rows of 2, 3 rows of 10, 10 rows of 3, 5 rows of 6, or 6 rows of 5.

Study Tip
When making an organized list of factor pairs, stop finding pairs when the factors begin to repeat.

On Your Own

Now You're Ready
Exercises 8–15

List the factor pairs of the number.

1. 18 **2.** 24 **3.** 51

4. **WHAT IF?** The woodwinds section of the marching band has 38 members. Which has more possible arrangements, the brass section or the woodwinds section? Explain.

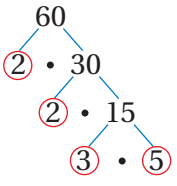
Key Idea

Remember
A *prime number* is a whole number greater than 1 with exactly two factors, 1 and itself. A *composite number* is a whole number greater than 1 with factors other than 1 and itself.

Prime Factorization

The **prime factorization** of a composite number is the number written as a product of its prime factors.

You can use factor pairs and a **factor tree** to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product. A factor tree for 60 is shown.



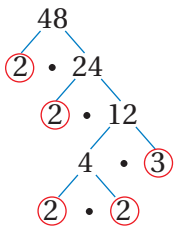
$60 = 2 \cdot 2 \cdot 3 \cdot 5$, or $2^2 \cdot 3 \cdot 5$

EXAMPLE 2 Writing a Prime Factorization

Write the prime factorization of 48.

Choose any factor pair of 48 to begin the factor tree.

Tree 1

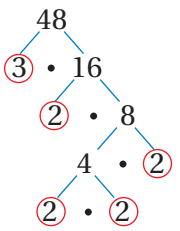


Find a factor pair and draw "branches."

Circle the prime factors as you find them.

Find factors until each branch ends at a prime factor.

Tree 2



$48 = 2 \cdot 2 \cdot 3 \cdot 2 \cdot 2$

$48 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

The prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, or $2^4 \cdot 3$.

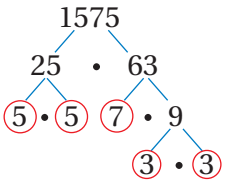
Study Tip

Notice that beginning with different factor pairs results in the same prime factorization. Every composite number has only one prime factorization.

EXAMPLE 3 Using a Prime Factorization

What is the greatest perfect square that is a factor of 1575?

Because 1575 has many factors, it is not efficient to list all of its factors and check for perfect squares. Use the prime factorization of 1575 to find any perfect squares that are factors.



$1575 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7$

The prime factorization shows that 1575 has three factors other than 1 that are perfect squares.

$3 \cdot 3 = 9$ $\quad$ $5 \cdot 5 = 25$ $\quad$ $(3 \cdot 5) \cdot (3 \cdot 5) = 15 \cdot 15 = 225$

So, the greatest perfect square that is a factor of 1575 is 225.

On Your Own

Now You're Ready
Exercises 16–23 and 29–32

Write the prime factorization of the number.

5. 20 **6.** 88 **7.** 90 **8.** 462

9. What is the greatest perfect square that is a factor of 396? Explain.

1.4 Exercises

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Vocabulary and Concept Check

1. **VOCABULARY** What is the prime factorization of a number?
2. **VOCABULARY** How can you use a factor tree to help you write the prime factorization of a number?
3. **WHICH ONE DOESN'T BELONG?** Which factor pair does not belong with the other three? Explain your reasoning.

2, 28	4, 14	6, 9	7, 8

Practice and Problem Solving

Use divisibility rules to determine whether the number is divisible by 2, 3, 5, 6, 9, and 10. Use a calculator to check your answer.

4. 1044 **5.** 1485 **6.** 1620 **7.** 1709

List the factor pairs of the number.

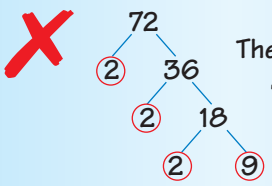
1 **8.** 15 **9.** 22 **10.** 34 **11.** 39

12. 45 **13.** 54 **14.** 59 **15.** 61

Write the prime factorization of the number.

2 **16.** 16 **17.** 25 **18.** 30 **19.** 26

20. 84 **21.** 54 **22.** 65 **23.** 77



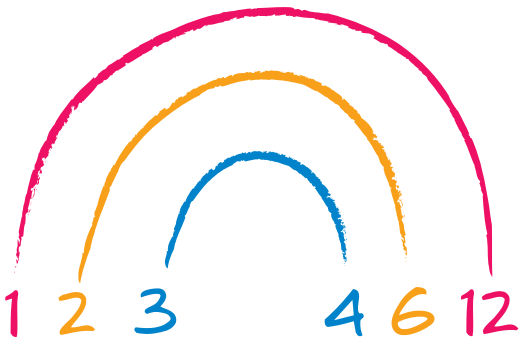
The prime factorization of

$72 = 2 \cdot 2 \cdot 2 \cdot 9$

$= 2^3 \cdot 9.$

24. ERROR ANALYSIS Describe and correct the error in writing the prime factorization.

25. FACTOR RAINBOW You can use a factor rainbow to check whether a list of factors is correct. To create a factor rainbow, list the factors of a number in order from least to greatest. Then draw arches that link the factor pairs. For perfect squares, there is no connecting arch in the middle. So, just circle the middle number. A factor rainbow for 12 is shown. Create factor rainbows for 6, 24, 36, and 48.



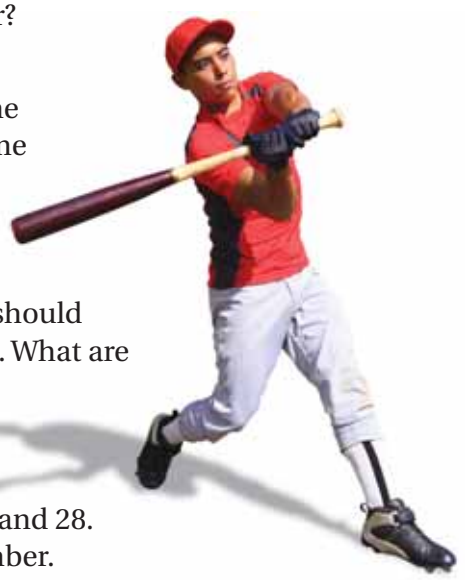
Find the number represented by the prime factorization.

26. $2^2 \cdot 3^2 \cdot 5$ **27.** $3^2 \cdot 5^2 \cdot 7$ **28.** $2^3 \cdot 11^2 \cdot 13$

Find the greatest perfect square that is a factor of the number.

3 **29.** 244 **30.** 650 **31.** 756 **32.** 1290

33. **CRITICAL THINKING** Is 2 the only even prime number? Explain.



34. **BASEBALL** The coach of a baseball team separates the players into groups for drills. Each group has the same number of players. Is the total number of players on the baseball team *prime* or *composite*? Explain.

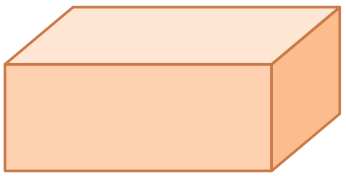
35. **SCAVENGER HUNT** A teacher divides 36 students into equal groups for a scavenger hunt. Each group should have at least 4 students but no more than 8 students. What are the possible group sizes?

36. **PERFECT NUMBERS** A *perfect number* is a number that equals the sum of its factors, not including itself. For example, the factors of 28 are 1, 2, 4, 7, 14, and 28. Because $1 + 2 + 4 + 7 + 14 = 28$, 28 is a perfect number. What are the perfect numbers between 1 and 28?

37. **BAKE SALE** One table at a bake sale has 75 cookies. Another table has 60 cupcakes. Which table allows for more rectangular arrangements when all the cookies and cupcakes are displayed? Explain.

38. **MODELING** The stage manager of a school play creates a rectangular acting area of 42 square yards. String lights will outline the acting area. To the nearest whole number, how many yards of string lights does the manager need to enclose this area?

Rectangular Prism



Volume = 40 cubic inches

39. **Volume** The volume of a rectangular prism can be found using the formula *volume* = *length* × *width* × *height*. Using only whole number dimensions, how many different prisms are possible? Explain.

Fair Game Review What you learned in previous grades & lessons

Find the difference. *(Skills Review Handbook)*

40. $192 - 47$ **41.** $451 - 94$ **42.** $3210 - 815$ **43.** $4752 - 3504$

44. **MULTIPLE CHOICE** You buy 168 pears. There are 28 pears in each bag. How many bags of pears do you buy? *(Skills Review Handbook)*

Ⓐ 5 Ⓑ 6 Ⓒ 7 Ⓓ 28